# Supporting Information

## S1. Time Evolution

Figure S1 shows the time evolution of the three selected cases. The BL is well-mixed in $\theta_{li}$ and $q_t$, and gradually becomes cooler and drier with time. In the reference and cold cases, cloud thinning is driven by the steady decrease of the cloud-top height, while the cloud base descends at a lower rate. In the warm case, while the cloud base ascends, the cloud top reaches its maximum at the 10th hour, then gradually descends until the cloud thins out.

The lifetime of clouds in ISDAC-i varies from 31 to 50 hours with $\mathscr{H}_f = 60\,\%$. Increasing $\mathscr{H}_f$ to 70% extends the cloud lifetime by about 10 to 20 hours (Figure S2). Because the lifetime depends on cloud thickness, it varies with cloud-top entrainment and large-scale subsidence rates. It also depends on the initial thickness of the cloud, which only varies with uniform temperature change. We see higher sensitivity to uniform temperature change in cases with strong temperature inversion.

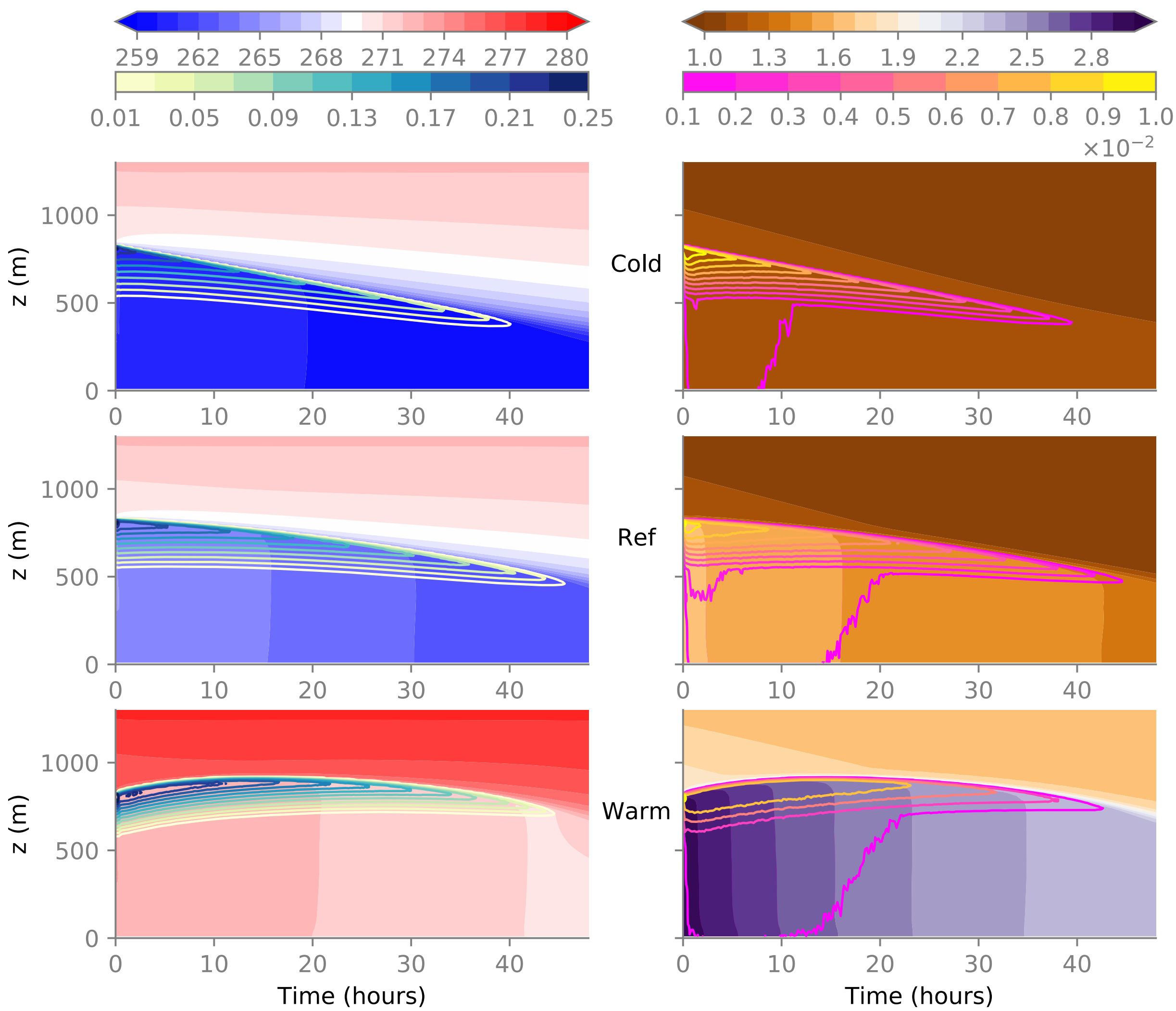


Figure S1. Time evolution of the ISDAC-i domain mean vertical structures of (left) liquid-ice potential temperature in colors, liquid water specific humidity in contours, and (right) total water specific humidity in colors, sum of cloud ice and snow specific humidity in contours. The cold, reference, and warm cases with $H_f = 60\%$ are shown from top to bottom. See main text for explanations of the cases.

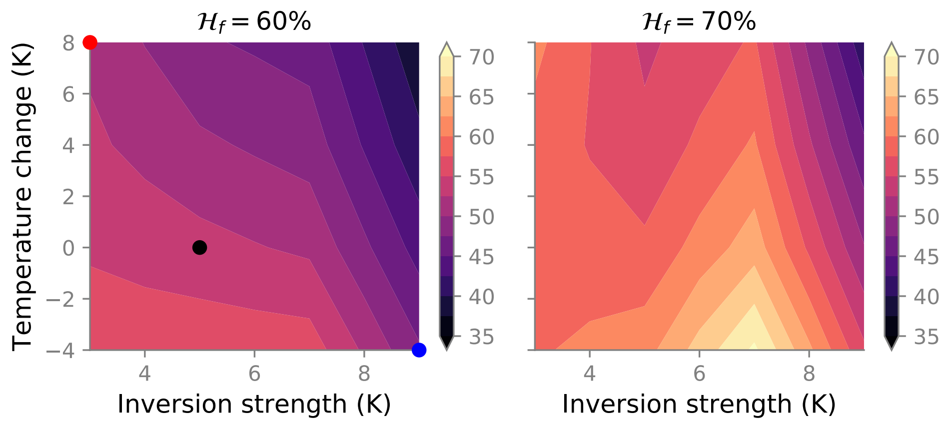


Figure S2: ISDAC-i cloud lifetime (in hours) from simulations with (left) $H_f$ = 60% and (right) $H_f$ = 70%. The horizontal axis shows the inversion strength, and the vertical axis shows the uniform temperature change relative to the reference simulation. The three dots correspond to the (red) warm, (black) reference, and (blue) cold cases.

## S2. ISDAC-i MLM results

Figures S3 and S4 are the MLM analogs of Figures 6 and 7. Figure S5 compares the magnitudes of LWP, entrainment rates, and cloud properties in the two models. The most significant difference is in LWP, where the magnitudes vary little for moderate and weak inversions (Figure S3). Most of the LWP gradients occur at cases with stronger inversions. LWP decreases with inversion strength and is insensitive to temperature change for cases with stronger inversion ($\Delta\theta_{li} \geq 7$ K). The minimal LWP occurs at $\Delta\theta_{li} = 9$ K, $\theta_{li,0} = 273$ K in the MLM, whereas for LES it occurs at $\Delta\theta_{li} = 9$ K, $\theta_{li,0} = 261$ K.

The magnitude of LWP is significantly higher in the MLM than in the LES (Figure S5a). Given that the points lie parallel to the 1:1 line, the difference in LWP between the two models may be taken as a constant offset in LWP. The higher condensate amount in MLM can be attributed to thicker cloud layers (Figure S4 and Figure S5f). On average, clouds are ~50 m thicker in the MLM than in the LES. This thick bias comes from a high bias in cloud-top height for weak inversions, and from a low bias in cloud base for moderate and strong inversions (Figure S5d and e). There is also a bias in $z_{base}$, mostly due to a mismatch in $q_t$. The near-surface precipitation flux in the LES is of magnitude comparable to that of the cloud-top entrainment in removing

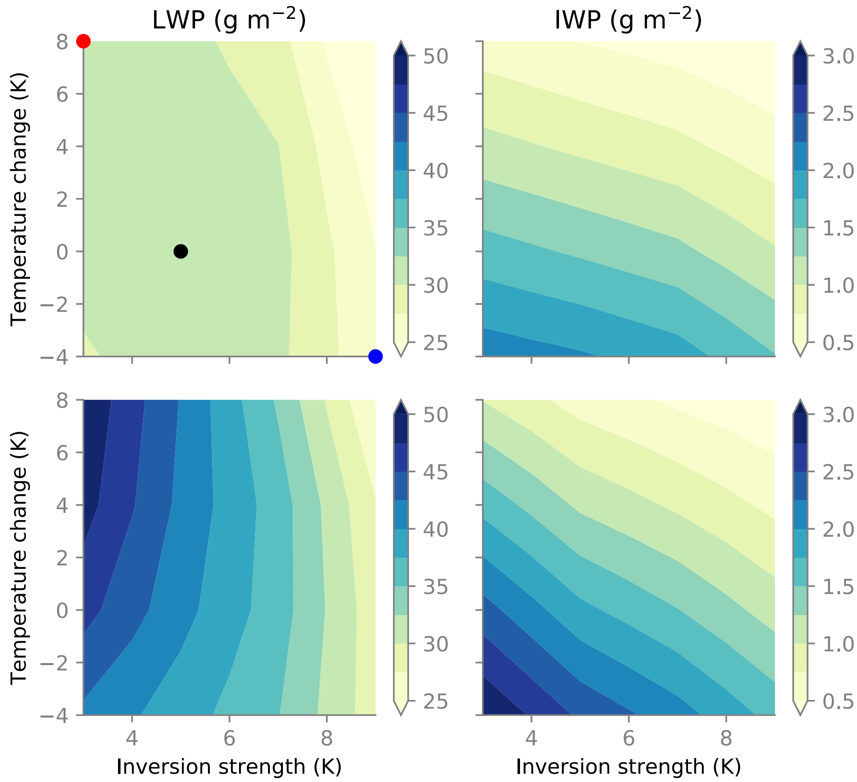


Figure S3: LWP and IWP in MLM ISDAC-i simulations averaged over the 24th hour. The horizontal axis shows the inversion strength, and the vertical axis shows uniform temperature changes. Top panels show simulations with $\mathscr{H}_f = 60\%$, and bottom panels show simulations with $\mathscr{H}_f = 70\%$. The three dots correspond to the (red) warm, (black) reference, and (blue) cold cases.

moisture out of the BL, especially at higher $\mathscr{H}_f$ (not shown). Given that precipitation is not represented in the MLM, we expect an overestimation of BL $q_t$, which is indeed what we see.

Differences in cloud-top entrainment also contribute to the mismatch of $q_t$ and $z_c$. In the MLM, everything above $z_i$ is prescribed and does not vary with time. In the LES, processes such as radiation and subsidence can change the profiles of $\theta_{li}$ and $q_t$ above the cloud top up to 1200 m. Above 1200 m, $\theta_{li}$ and $q_t$ are nudged toward the initial profiles. The lapse rate right above the cloud top is smoothed due to radiative cooling, especially for cases with a strong inversion. Therefore, $\Delta\theta_{li}$ in MLM is slightly biased high compared to LES (Figure S5c). This further leads to weaker entrainment drying and a high bias in MLM $q_t$.

There are also differences in the cloud top radiative flux jump in the two models that lead to different entrainment rates (Figure S5c). Because clouds are biased thick, the cloud layers do not dissipate in most of the cases in the MLM. If we artificially correct the entrainment rate towards LES values, we start to see cloud dissipation and similar cloud lifetimes in the MLM and LES.

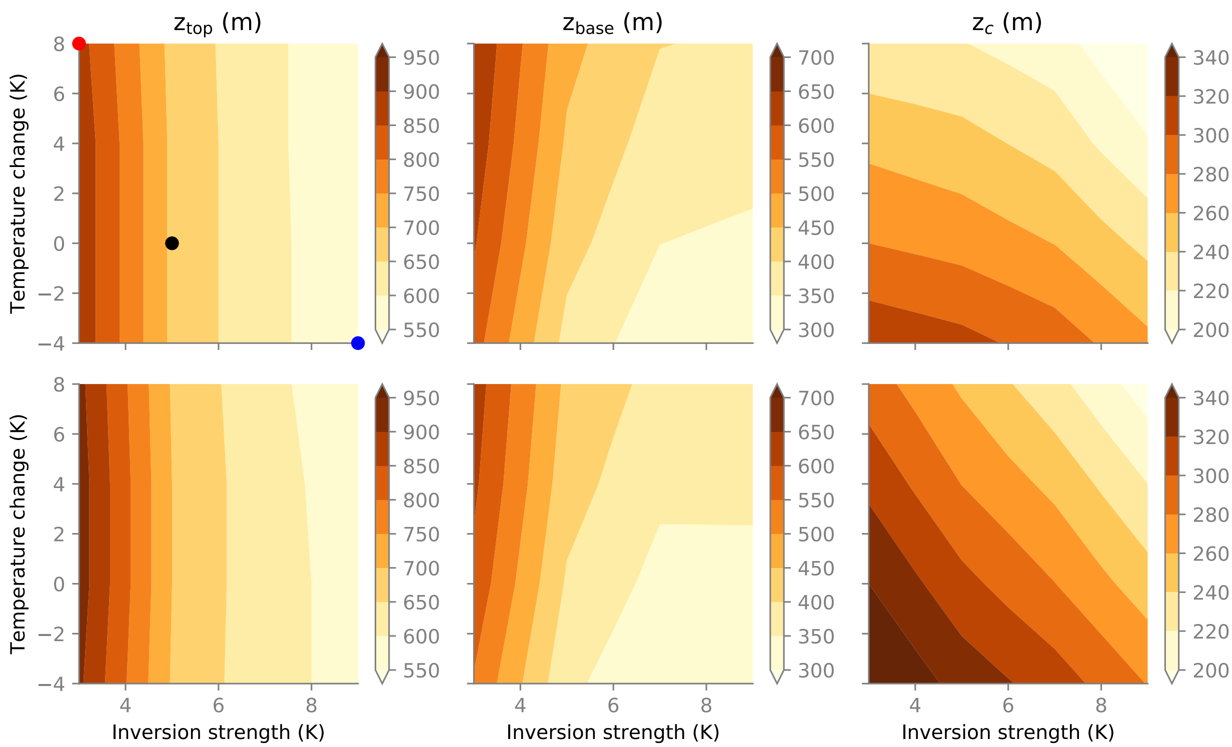


Figure S4: Same as Figure S3, but for MLM cloud-top height $z_{top}$, cloud-base height $z_{base}$, and cloud thickness $z_c$.

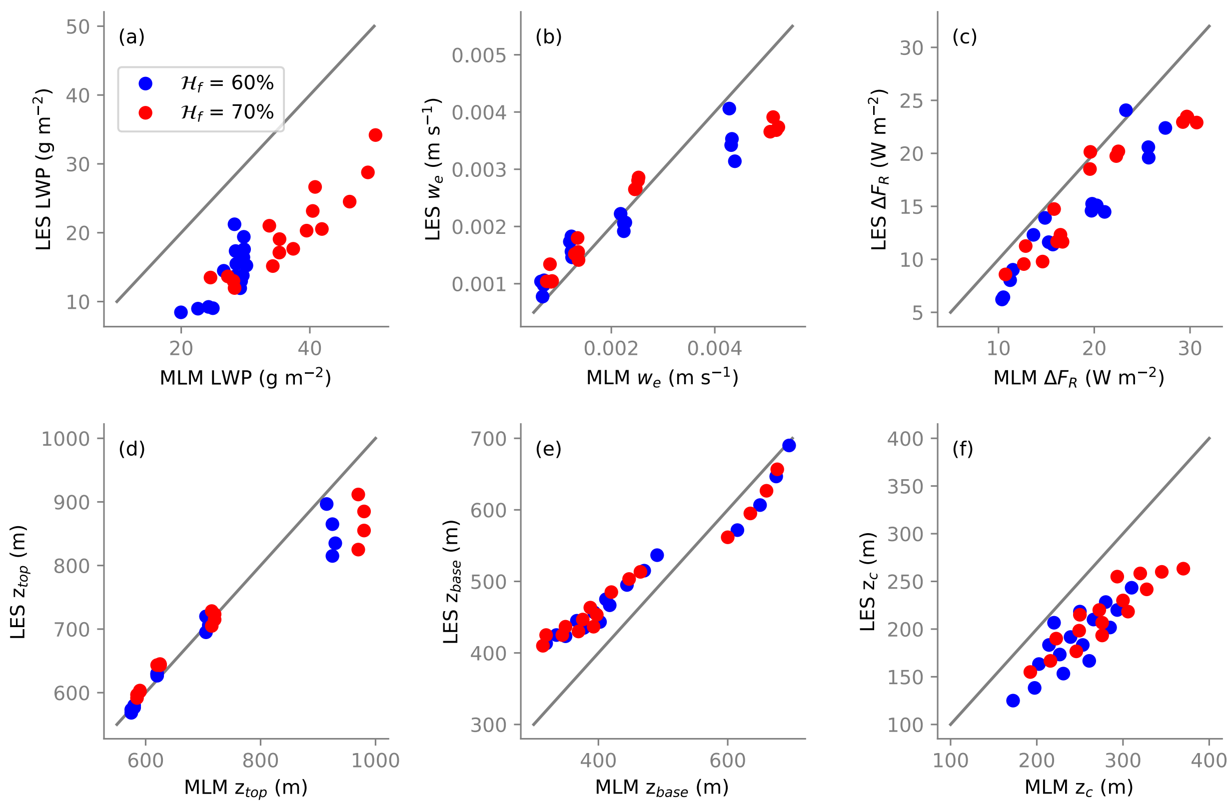


Figure S5: Scatter plots of (a) LWP, (b) entrainment rate, (c) radiative flux jump at cloud top, (d) cloud-top height, (e) cloud-base height, and (f) entrainment rate, from ISDAC-i simulations averaged at the 24th hour from MLM (horizontal axis) and LES (vertical axis). The gray lines indicate the 1:1 line.